Digital

7/3/2016 الاثنين د. عرفة محاضرة [4]

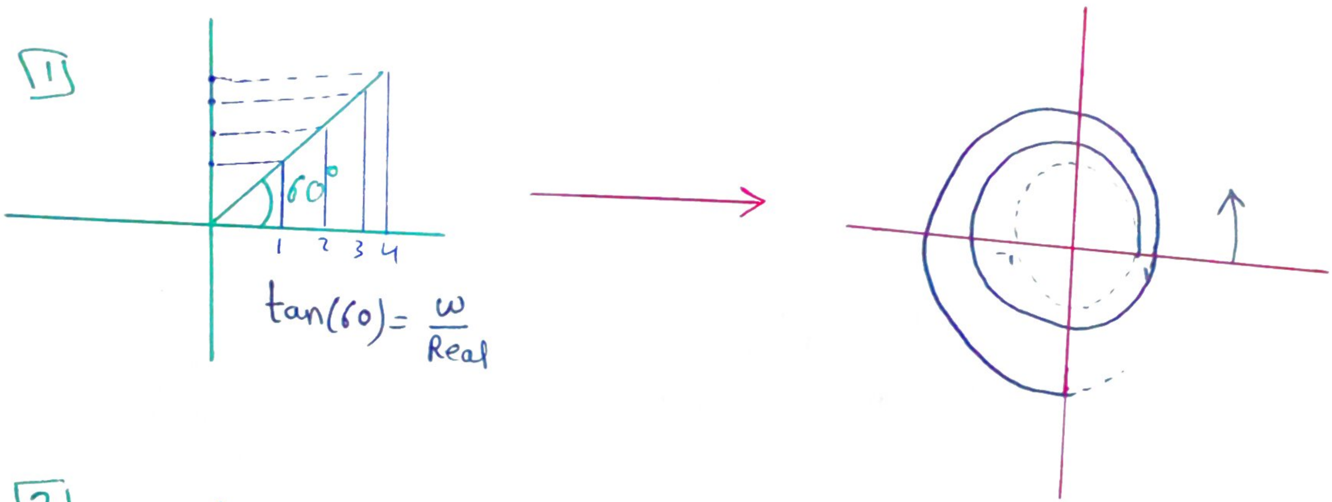
Remember:

$$S = \sigma + j\omega \Longrightarrow z = e^{TS} = |\;| \angle\_\_ = r \angle \theta$$

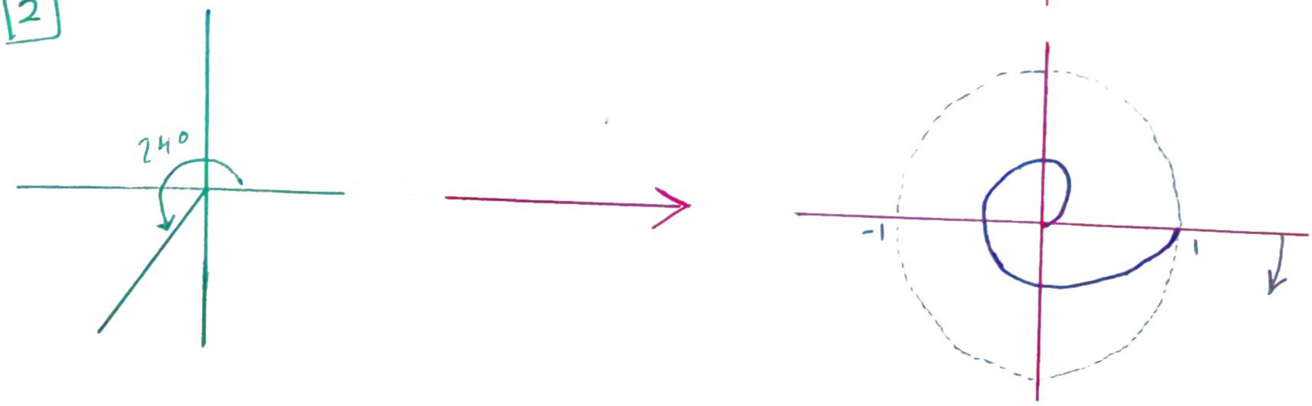
$$|\;| = r = e^{\sigma T} \quad \& \quad \theta = \omega T$$

Quiz Solution

[1]



[2]



Review Mapping from Digital.Lec.03.pdf

Pages: 11, 12, 13

Check system Stability :

[1] poles Location :-

ex: ch. eqn

$$z^2 - z + 0.5 = 0$$

poles $\Rightarrow z_{1,2} = 0.5 \pm j0.5$

$$|z_{1,2}| = \sqrt{(0.5)^2 + (0.3)^2} = 0.767 < 1 \quad [\text{Stable}]$$

ex2: ch. eqn

$$(z-0.5)(z+0.7)(z-1) = 0$$

poles $\Rightarrow z_{1,2,3} = 0.5, -0.7, 1$

$$|z_{1,2,3}| = 0.5, \ 0.7, \ 1$$

$<1 \quad <1 \quad = 1$

Critically Stable

ex3: ch. eqn

$$(z-0.5)(z+0.7)(z-1)(z+1.5) = 0$$

poles $\Rightarrow z_1 = 0.5, \ z_2 = -0.7, \ 1, \ z_4 = -1.5$

$$|z_{1,2,3,4}| = 0.5, \ 0.7, \ 1, \ 1.5$$

Stable Critical unstable

$\therefore$ the system is unstable

$\Longrightarrow$ Continue

[2] Using bilinear transformation

$$Z = e^{TS} = e^{T/2\, S} \cdot e^{T/2\, S}$$

$$= \frac{e^{T/2\, S}}{e^{-T/2\, S}}$$

$$e^{x} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

* for small values of x $\Rightarrow e^{x} \simeq 1 + x$

$$Z = \frac{1 + \frac{T}{2} S}{1 - \frac{T}{2} S}$$

* assume $\frac{T}{2} S = r$

$$Z = \frac{1+r}{1-r}$$ (stability can be checked using routh method)

Ex: ch. equation

$$Z^3 + 3.3 Z^2 + 3Z + 0.8 = 0$$

Check system stability using bilinear transformation

$$Z = \frac{1+r}{1-r}$$

$$\left(\frac{1+r}{1-r}\right)^3 + 3.3\left(\frac{1+r}{1-r}\right)^2 + 3\left(\frac{1+r}{1-r}\right) + 0.8 = 0$$

Multiply by $(1-r)^3$

$$(1+r)^3 + 3.3(1+r)(1-r) + (1+r)(1-r)^2 + 0.8(1-r)^3 = 0$$

$$(1+r)(1+2r+r^2) + 3.3(1+2r+r^2)(1-r) + 3(1+r)(1-2r+r^2) + 0.8(1-r)(1-2r+1) = 0$$

$$\Rightarrow \boxed{r^3 + 9r^2 - 9r - 81 = 0}$$ (ch. eqn in r-domain)

using routh method

r^3	1	-9
r^2	9	-81
A(r) ~~r^1~~	~~0~~	
r^1	18	
r^0	-81	

$$A(r) = 9r^2 - 81$$

$$\frac{dA(r)}{dr} = 18r$$

unstable since there is a change in sign

using calculator $z_{1,2,3} = -2, -0.5, -0.8$

$$|z_{1,2,3}| = 2, \ 0.5, \ 0.8$$

↳ unstable

Exe: $z^3 - 0.2z^2 - 0.25z + 0.05 = 0$

$$z = \frac{1+r}{1-r}$$

ch. eqn $\Rightarrow$

$$\left(\frac{1+r}{1-r}\right)^3 - 0.2\left(\frac{1+r}{1-r}\right)^2 - 0.25\left(\frac{1+r}{1-r}\right) + 0.05 = 0$$

$$(1+r)^3 - 0.2(1+r)^2(1-r) - 0.25(1+r)(1-r)^2 + 0.05(1-r)^3 = 0$$

$$0.9r^2 + 3.6r^2 + 2.9r + 0.6 = 0$$

r^3	0.9	2.9
r^2	3.6	0.6
r^1	2.75	
r^0	0.6	

there is no sign changes in first column $\Rightarrow$ stable system

$\Longrightarrow$ Continue

3 Using Jury test

* assume that the ch. equation $F(z) = 1 + \overline{GH}(z) = 0$
* in general form ch. eq^{n}

$$1 + \overline{GH}(z) = F(z) = a_n z^n + a_{n-1} z^{n-1} + a_{n-2} z^{n-2} + \dots + a_1 z + a_0 = 0$$

for the system to be stable:

① $F(1) > 0$

② $(-1)^n F(-1) > 0$

$n \rightarrow$ system order

③ $|a_0| < |a_n|$

} Sufficient for 2nd order systems

— For $n > 2$

Construct Jury Array

row no.	z^0	z^1	z^2 ----	z^{n-1}	z^n
1	a_0	a_1	a_2 ---	a_{n-1}	a_n
2	a_n	a_{n-1}	a_{n-2} ---	a_1	a_0
3	b_0	b_1	b_2 ----	b_{n-1}	
4	b_{n-1}	b_{n-2}	b_{n-3} ----	b_0	
$2n-3$ آخر صف	r_0	r_1	r_2	ثلاث عناصر فقط	

عكس ترتيب

عكس ترتيب

عدد العناصر تقل بمقدار ①

$\Longrightarrow$ continue

ثابت

$$b_0 = \begin{vmatrix} a_0 & a_n \\ a_n & a_0 \end{vmatrix} \quad ; \quad b_1 = \begin{vmatrix} a_0 & a_{n-1} \\ a_n & a_1 \end{vmatrix}$$

عكس الجميع — الثاني من اليسار — الثاني من اليمين

$$b_2 = \begin{vmatrix} a_0 & a_{n-2} \\ a_n & a_2 \end{vmatrix}$$

④ $|b_0| \overset{?}{>} |b_{n-1}|$ القيمة الأولى والأخيرة

⑤ $|r_0| > |r_2|$

Ex:- ch. eqn : $F(z) = z^3 - 1.8z^2 + 1.05z - 0.2 = 0$

① $F(1) > 0$?

$= 1 - 1.8 + 1.05 - 0.2 = 0.05 > 0$ ✓

② $(-1)^{n=3} F(-1) > 0$?

$\Rightarrow -(-1 - 1.8 - 1.05 - 0.2) > 0$ ✓

③ $|a_0| < |a_3|$?

$0.2 < 1$ ✓

	z^0	z^1	z^2	z^3
1	-0.2	1.05	-1.8	1
2	1	-1.8	-1.05	-0.2
3	b_0	b_1	b_2	ثلاث عناصر فقط

$$b_0 = \begin{vmatrix} -0.2 & 1 \\ 1 & -0.2 \end{vmatrix} = -.96$$

$$b_1 = \begin{vmatrix} -0.2 & -1.8 \\ 1 & 1.05 \end{vmatrix} = -1.59 \longleftarrow$$ مش محتاجها

$$b_2 = \begin{vmatrix} -0.2 & 1.05 \\ 1 & -1.8 \end{vmatrix} = -0.69$$

(4) $|b_0| > |b_2|$?

$0.96 > 0.69$ ✓

the system is stable

Ex2: ch. equation

$$F(z) = z^4 - 1.7z^3 + 1.04z^2 - 0.268z + 0.024 = 0$$

$\boxed{1}$ $F(1) > 0$? ✓

0.096

$\boxed{2}$ $(-1)^{n=4}\, F(-1) > 0$? ✓

$F(-1) > 0$ ✓✓

$\boxed{3}$ $|a_0| = 0.024 < |a_4| = 1$ ✓

	z^0	z^1	z^2	z^3	z^4
1	0.024	−0.268	1.04	−1.7	1
2	1	−1.7	1.04	−0.268	0.024
3	b_0	b_1	b_2	b_3	
4	b_3	b_2	b_1	b_0	
5	c_0	c_1	c_2		

$\Longrightarrow$ continue

EX2: Ch. eq$^{\underline{n}}$

$$b_0 = \begin{vmatrix} 0.024 & 1 \\ 1 & 0.024 \end{vmatrix} = -0.999 \simeq -1$$

$$b_1 = \begin{vmatrix} 0.024 & -1.7 \\ 1 & -0.268 \end{vmatrix} = 1.693$$

$$b_2 = \begin{vmatrix} 0.024 & 1.04 \\ 1 & 1.04 \end{vmatrix} = 1.015$$

$$b_3 = \begin{vmatrix} 0.024 & -0.268 \\ 1 & -1.7 \end{vmatrix} = 0.227$$

$\boxed{4}$ $|b_0| = 1 > |b_3| = 0.227$? ✓

$$C_0 = \begin{vmatrix} b_0 = -1 & 0.227 \\ b_3 = 0.227 & -1 \end{vmatrix} = 0.943$$

$$C_2 = \begin{vmatrix} -1 & 1.693 \\ 0.227 & -1.015 \end{vmatrix} = 0.63$$

$\boxed{5}$ $|C_0| = 0.943 > |C_2| = 0.63$? ✓

System is stable

⟹ continue

Ex3: ch. equation

$$F(z) = z^3 + 3.3z^2 + 3z + 0.8 = 0$$

$\boxed{1}$ $F(1) > 0$? ✓

$\boxed{2}$ $(-1)^{n=3} F(-1) > 0$?

$-(-1 + 3.3 - 3 + 0.8) > 0$? X

System is unstable